Higgsless W Unitarity from Decoupling Deconstruction

Nick Evans and Phil Membry [*]

School of Physics and Astronomy
Southampton University
Southampton, S017 1BJ
United Kingdom

Abstract

Recently there has been interest in electroweak models on a five dimensional interval that break the symmetry without a higgs boson. By warping the metric of the interval it may be possible to avoid experimental bounds on extra W bosons and $\delta\rho$. Five dimensional models necessarily require an explicit UV cut-off to remain perturbative, such as that provided by deconstruction. We study a simple deconstruction of this scenario with a chain of $SU(2)^{N+1} \times U(1)_Y$ groups linked by bi-fundamental higgs. There are two interesting decoupling limits of this model, when the higgs vevs are taken large and when the SU(2) couplings grow, which might provide a perturbative realization. In fact it is very challenging to satisfy all the experimental bounds and the most compatible scenario has both a higgs and a relatively strongly coupled new W both close to 2 TeV.

---

[*]`evans@phys.soton.ac.uk, pjm@phys.soton.ac.uk`

arXiv:hep-ph/0406285v2 11 Aug 2004

The need to restore unitarity in high energy WW scattering has long been cited as evidence that there must be a higgs boson with mass below of order 1 TeV [1]. Recently though it has been realized that unitarity can also be restored by a Kaluza Klein (KK) like tower of massive W-bosons without a higgs [2, 3, 4, 5]. These models [4]-[20]are variants on the idea that there is a fifth dimension that is a discrete interval. The gauge group is broken by boundary conditions at the ends of the interval rather than by a higgs mechanism. In the four dimensional theory at long distance scales there are only the W, Z fields and their KK towers, yet the theory is unitary.

Such a model must though meet the stringent experimental constraints on the masses of extra W bosons, and on the precision data for $\sin \theta_W$ and $\delta \rho$ (or equivalently the parameters S and T). Some of the above models in which the extra dimension is warped have made progress in meeting these constraints. However a five dimensional theory is necessarily ill defined in the UV where it becomes strongly coupled and must have some UV completion before strong coupling is reached. One must be careful not to make use of spacetime curvature on scales where the theory is strongly coupled - the AdS metric used in [5] with an exponential warp factor may for example be hard to support. Also the analysis of [12], which uses the models we study below, explicitly works in the strong coupling limit.

To keep track of the gauge coupling strength it is useful to have a fully defined theory with an explicit UV completion. Deconstruction [21, 22] provides such a realization with the fifth dimension manually constructed by the reproduction of the Kaluza Klein tower in a renormalizable four dimensional theory. The extra fifth dimension is first thought of as a lattice where a separate copy of the four dimensional gauge group lives at each site. The sites are then linked by Goldstone fields transforming in the $(N, \bar{N})$ representation of the two neighbouring site gauge groups. The resulting gauge boson mass spectrum, in the purely four dimensional model, then mimics a KK tower at scales well below the symmetry breaking scale. A fully renormalizable gauge theory can be found by promoting the Goldstone fields to a full higgs multiplet. WW unitarity is restored by the Kaluza Klein tower at low energies and finally at the very high fundamental symmetry breaking scale by the higgs bosons [2]. These models therefore are only higgsless in the sense that the higgs mass rises relative to that of the Standard Model and phenomenology may appear higgsless at the LHC.

The simplest deconstruction extension of the Standard Model has been suggested by a number of authors [12, 15, 16, 13]. It consists of multiple repeats of the SU(2) gauge group as shown in the moose diagram notation [23, 24] of Fig 1. There are $N+1$ copies of SU(2) each potentially with a unique coupling g_i. The gauge bosons are coupled by bi-fundamental higgs with vevs v_i linking SU(2)$_i$ and SU(2)$_{i+1}$. Finally the $(N + 1)$th SU(2) is coupled by the $(N + 1)$th higgs to a U(1)$_Y$ hypercharge group. This final symmetry breaking pattern ensures that there is a
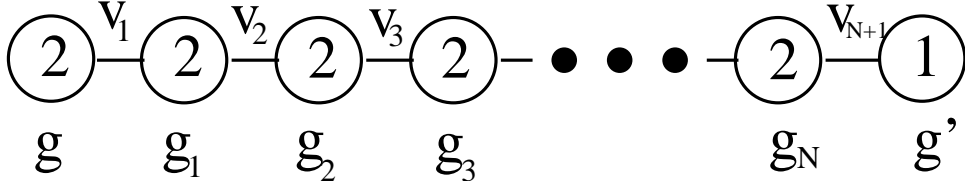
massless photon.

Figure 1: The moose model under consideration - numbered circles represent SU(N) gauge groups and links bi-fundamental higgs fields.

The low energy dynamics is described by a non-linear realization of the Goldstone fields

$$\mathcal{L} = \sum_i \frac{v_i^2}{4} Tr D^\mu U_i^\dagger D_\mu U_i + \text{higher derivative} \tag{1}$$

where as usual $U_i = exp(2i\pi_i^a T^a/v_i)$ with π_i^a the Goldstone fields associated with the broken generators T^a. The gauge fields enter the covariant derivatives with generators acting on U_i from the left or right depending upon their coupling to the left or right in the moose diagram.

The tree level W and Z mass matrices may be read off as

$$M_{Wij}^2 = \begin{pmatrix} g^2v_1^2 & -gg_1v_1^2 & 0 & 0 & ... & 0 & 0 & 0 \\ -gg_1v_1^2 & g_1^2(v_1^2+v_2^2) & -g_1g_2v_2^2 & 0 & ... & 0 & 0 & 0 \\ 0 & -g_1g_2v_2^2 & -g_2^2(v_2^2+v_3^2) & -g_2g_3v_3^2 & ... & 0 & 0 & 0 \\ . & . & . & . & ... & . & . & . \\ . & . & . & . & ... & . & . & . \\ 0 & 0 & 0 & 0 & ... & 0 & -g_{N-1}g_Nv_{N-1}^2 & g_N^2(v_{N-1}^2+v_N^2) \end{pmatrix} \tag{2}$$

$$M_{Zij}^2 = \begin{pmatrix} g^2v_1^2 & -gg_1v_1^2 & 0 & 0 & ... & 0 & 0 & 0 & 0 \\ -gg_1v_1^2 & g_1^2(v_1^2+v_2^2) & -g_1g_2v_2^2 & 0 & ... & 0 & 0 & 0 & 0 \\ 0 & -g_1g_2v_2^2 & -g_2^2(v_2^2+v_3^2) & -g_2g_3v_3^2 & ... & 0 & 0 & 0 & 0 \\ . & . & . & . & ... & . & . & . & . \\ . & . & . & . & ... & . & . & . & . \\ 0 & 0 & 0 & 0 & ... & 0 & -g_{N-1}g_Nv_{N-1}^2 & g_N^2(v_{N-1}^2+v_N^2) & g_Ng'v_N^2 \\ 0 & 0 & 0 & 0 & ... & 0 & 0 & g_Ng'v_N^2 & g'^2v_N^2 \end{pmatrix} \tag{3}$$

Note that in the limit where $N = 0$ this description of the Goldstone modes of the model is simply the Standard Model. In fact to completely recover the Standard Model the higgs in the UV completion must also be made real.

For larger N when the couplings and vevs are all equal the W mass matrix has eigenvalues [4]

$$M_W^k = gv \sin\left[\frac{(2k-1)\pi}{4N-2}\right] \tag{4}$$

3

which for large N and $k < N$ reproduces a KK like tower of W states. Note that the W tower masses are suppressed relative to v by a factor of N. This is the mechanism by which we will remove the higgs from the low energy spectrum. In fact the couplings g grow as $\sqrt{N}$ to keep the low energy coupling invariant so the gain in higgs mass is only $\sqrt{N}$ too. In a simple higgs model the higgs mass is given by $\sqrt{\lambda}v$ with λ the quartic coupling in the higgs potential. Thus as the higgs vev increases by a factor of $\sqrt{N}$ so does it's mass. In fact in the UV completion the scalar potential could be considerably more complicated with renormalizable terms of the form $|h_i|^2|h_j|^2$ affecting the masses, but it is only our intention here to study the dependence of the vev on N which is indicative of the higgs mass scale. Unitarity in W scattering must still be maintained at scales of order the lightest W mass - as discussed in [2] the KK tower acts as the restoring mechanism.

This simple set up will not make for good phenomenology since the first KK partner of the W is very light (the direct experimental bound is of order 500 GeV). We must therefore look at limits where the KK modes are starting to become more massive and decouple. There are two obvious limits of this form. Firstly we can raise the vevs $v_1 - v_N$; in the limit where they are infinite the low energy theory just becomes the Standard Model. This limit seems promising since precision parameters will naturally tend to the Standard Model values in this limit too (we will see soon how well the new physics decouples). However, the lightest higgs is becoming Standard Model like too in this limit and hence light. The second limit, explored in [12], is to take the couplings $g_1 - g_N$ to be large - this makes the KK modes heavy but does not precisely return the Standard Model even in the infinite coupling limit. Varying the vevs and couplings along the chain corresponds at the five dimensional level to warping the geometry [25] so we might hope to find the same successes seen in such models. We will explore both of these limits shortly.

To present results that can be compared to experimental data we will numerically solve for the eigenvalues of the matrices (2,3). At this stage we will simply work at tree level and search for a theory compatible with the data at this level. We must also couple the Standard Model matter fields into the model. We will follow [12] and allow the fermions to couple to the end two gauge groups in the moose chain. This choice ensures that $T = 0$ [12] when the central SU(2) groups' couplings are taken large. We have also explored other assignments but found little benefit from them. As usual we will fix our model to the measured values of M_Z the electric charge e and the Fermi constant G_F since these are the best measured experimental results.

Let us first study the limit where the gauge couplings of the internal groups of the moose are taken large. We fix $g_1...g_n = \tilde{g}$ and take couplings for the end groups g and g' as shown in Fig 1. We also keep all the scalar vevs equal for now. Fixing the parameters to the above experimental quantities is a little involved. For a given value of $\tilde{g}$ the Z mass matrix depends

on three parameters g, g', v and determines the two parameters M_Z, e. Numerically we first fix the physical value of e, which is given by $e = g\alpha_1 + g'\alpha_2$ where the αs are the relevant mixings, to fix g as a function of g'. The lowest mass eigenvalue of the Z mass matrix is then fixed to the physical Z mass giving us numerical values for g and g' as a function of v. Finally we diagonalize the W mass matrix and determine v by requiring the physical value of G_F which fixes each of g, g', v.
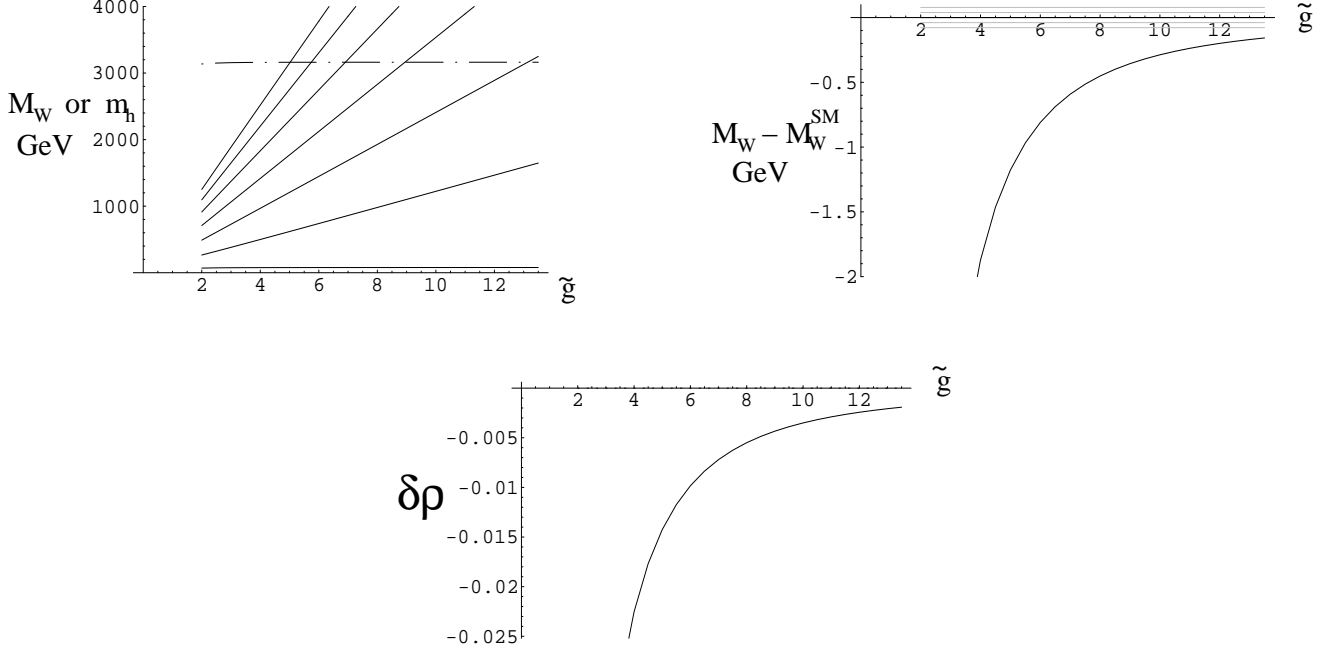

Figure 2: Results in the model with N=10. The first figure shows the tower of W boson states against changing $\tilde{g}$. The dot-dashed line is the maximum higgs mass in the model. The second figure shows how the lightest W boson mass deviates from the Standard Model value and the experimental constraints at 1 and 2 σ. The third figure shows $\delta\rho$ in the model - the experimental constraints is $\delta\rho \geq -0.001$.

We plot for the case where $N = 10$ the W and KK W masses as a function of $\tilde{g}$ in Fig 2. We also plot the maximum higgs mass which we take to be a factor of $1TeV/(125)GeV$ times the higgs vev (thus we would say the Standard Model higgs must lie below 1TeV). The first success is that the higgs boson mass rises by a factor of 3 relative to the Standard Model. The KK modes also become more massive as $\tilde{g}$ grows. However, the second figure showing just the W mass and the experimental bounds reveals that the model is some way off the data even as $\tilde{g}$ approaches 4π, an absolute maxmum value for perturbativity. Finally we have also plotted $\delta\rho = M_W^2/(M_Z^2 \cos\theta_w) - 1$ with $\cos\theta_w$ obtained by equating the electron coupling to the Z in our model to the Standard Model expression. We find a negative value of $\delta\rho$ that falls as $\tilde{g} \to \infty$ but

is still quite substantial at $\tilde{g} = 4\pi$. The relation between our definition of $\delta\rho$ and the parameter T is discussed in more detail in the appendix.

It's interesting to look at the N dependence of these models. As an example we plot in Fig 3 the W mass as a function of N when $\tilde{g} = 4\pi$. We see that increasing N actually moves the results away from the data (although of course the rise in the higgs mass grows as $\sqrt{N}$ so one would hope to strike a balance).
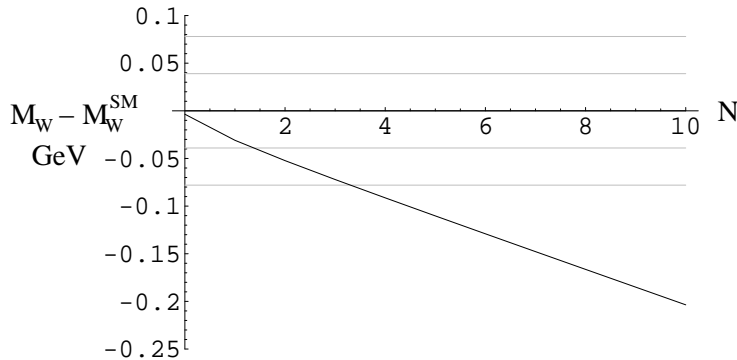
Figure 3: The dependence of M_W on N in the moose models with $\tilde{g} = 4\pi$.

We have though at our disposal a second decoupling limit for the KK modes. Consider again the moose model with $N = 10$. If we take all the vevs except one to infinity then the model returns to the Standard Model and all experimental constraints will be met! As an example of this in Fig 4 we plot the gauge boson masses in models with $\tilde{g} = 2$ and v_1 to $v_N = \tilde{v}$ and $v_{N+1} = v$. We plot the results for each value of $\tilde{v}/v$. The extra decoupling is again apparent as the KK Ws rise in mass. However, here we know that the higgs mass must fall to the Standard Model value as $\tilde{v}/v \to \infty$ and from Fig 4 we see that this fall in the higgs mass is much more dramatic than the rise induced in the KK tower - precisely the opposite of what one would require phenomenologically.
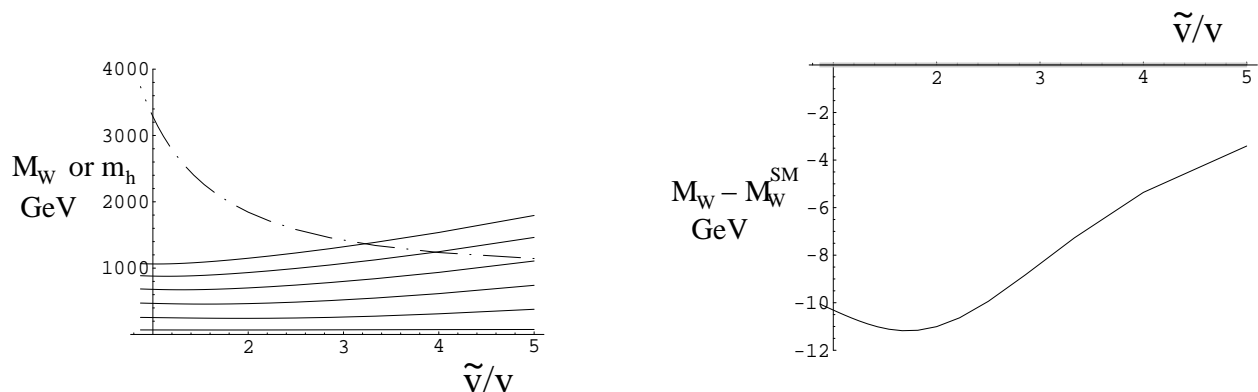
Figure 4: Results for the lightest Ws and higgs mass in the $N = 10$ model with $\tilde{g} = 2$ and for varying the vevs $\tilde{v}/v$. The right-hand side of the plot approaches the Standard Model limit.

6

Although these two limits are not themselves sufficient to provide an interesting model one might hope that some combination of the two scenarios might. As an example in the $N = 10$ model we can set $\tilde{g} = 4\pi$ and look at varying $\tilde{v}/v$. We plot the results in Fig 5 for M_W and m_h.
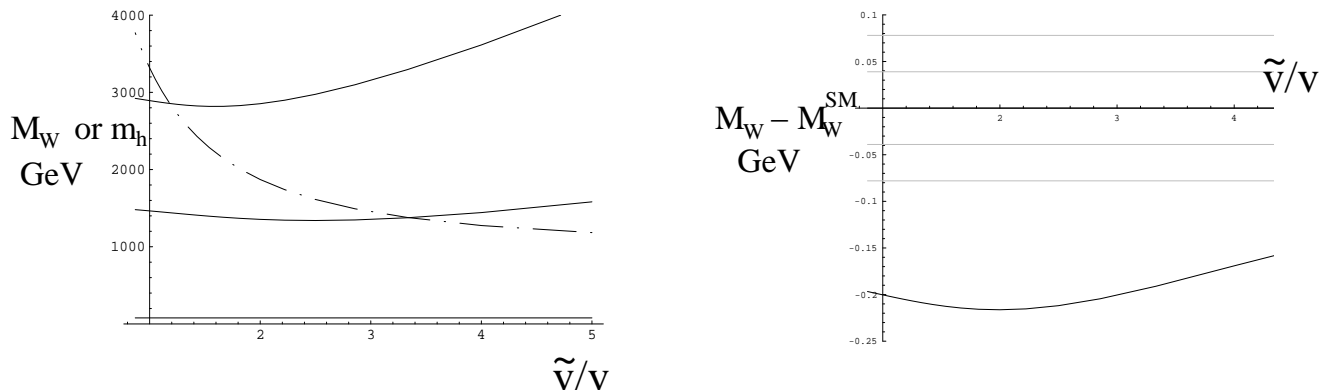

Figure 5: Results for the lightest Ws and higgs mass in the $N = 10$ model with $\tilde{g} = 4\pi$ and for varying the vevs $\tilde{v}/v$.

In fact in the area of parameter space where the higgs is heavy ($> 2TeV$) M_W is still some way off the data. We have tried numerically adjusting the vevs and couplings along the chain but have not found any significant improvement in the match to the data. In fact it is clear in this case that most of the heavy Ws are so heavy they do not influence low energy unitarity. For this reason it makes sense to restrict to models with small values of N which we saw above are in better accord with data. We do though want to see a gain in the higgs mass which requires large N - as a compromise lets consider $N = 4$ from which we can hope to obtain a rise in higgs mass of a factor of 2. We again set $\tilde{g} = 4\pi$ and plot results against $\tilde{v}/v$ in Fig 6.

Both M_W and $\delta\rho$ lie closer to the data when the higgs is somewhat heavier than in the Standard Model in this case. In fact though the next W in the tower of states lies close to 2 TeV in this model. This was not the intention for these models where it was hoped that the LHC would find heavy Ws and no higgs. Here we have identified, at best, the possibility that a combination of a higgs and quite strongly coupled W both at 2 TeV might viably restore unitarity (a similar conclusion is reached in [11] by an analysis of five dimensional models). The LHC might struggle to identify such a model. We conclude that finding a higgsless model in agreement with the precision data appears very hard - one would perhaps need at least some subtle symmetries in a model to achieve this goal.
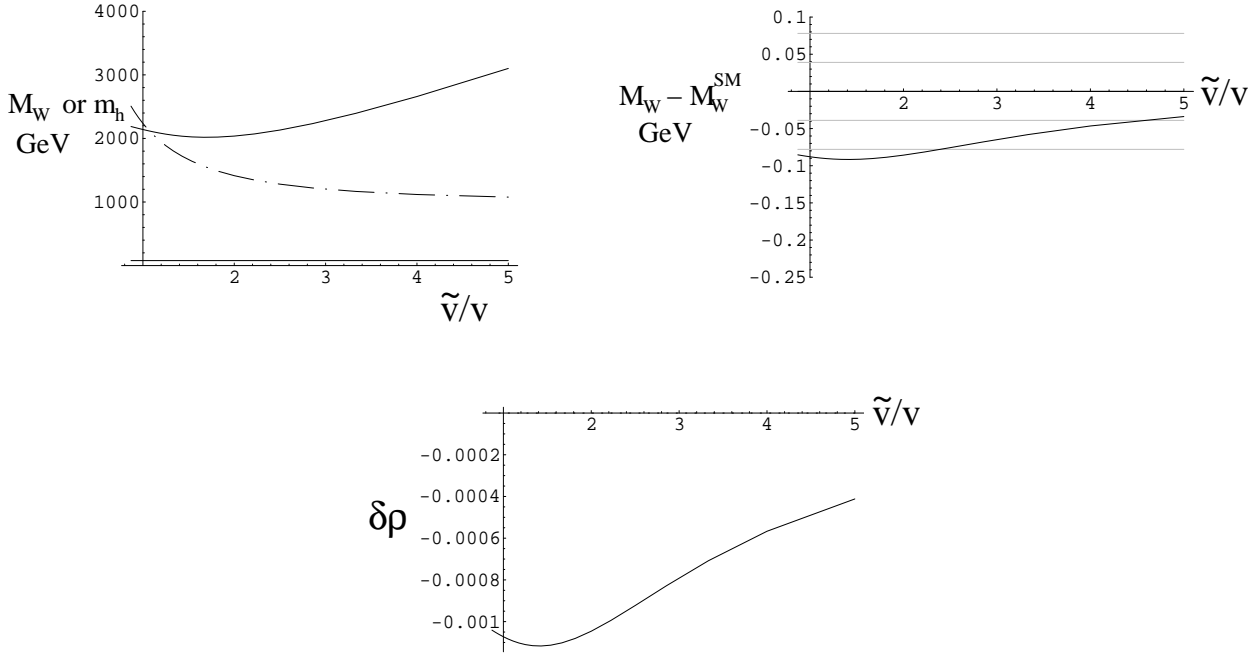
Figure 6: Results for the lightest Ws and higgs mass in the $N = 4$ model with $\tilde{g} = 4\pi$ and for varying the vevs $\tilde{v}/v$. We also show $\delta\rho$ vs $\tilde{v}/v$. Here the model lies close to the experimental bounds with a higgs of order 2 TeV.

Appendix

Since the release of this work as a preprint the authors of [17] have pointed out the following interesting relation between $\delta\rho$ as we define it and the parameters S, T. The models we study have been shown to have the parameters $T = U = 0$ and positive S in [15]. $\delta\rho$ can be defined either as we do in terms of the pole W and Z masses or as the ratio of the charged and neutral weak currents at low energies. This latter definition gives $\delta\rho = \alpha T$. The pole W and Z masses, given in terms of the parameters e^2, s^2, G_F and corrections S, T, U are given by [17]

$$M_Z^2 = \frac{1}{4\sqrt{2}G_f \left(\frac{s^2 c^2}{e^2} - \frac{S}{16\pi} + s^2 c^2 \frac{T}{4\pi} \right)} \tag{5}$$

$$M_W^2 = \frac{1}{4\sqrt{2}G_f \left(\frac{s^2}{e^2} - \frac{S+U}{16\pi} \right)} \tag{6}$$

So defining ρ as $M_W^2/M_Z^2 c^2$ gives

$$\delta\rho = \alpha T - \frac{\alpha S}{4c^2} + \frac{\alpha U}{4s^2} \tag{7}$$

Thus our negative values for $\delta\rho$ are consistent with and caused by the presence of positive S.

8

References

[1] C.H. Llewellyn Smith,. *High-Energy Behaviour and Gauge Symmetry*, Phys. Lett. **B46** (1973) 233.

[2] R.S. Chivukula, Hong-Jian He, *Unitarity of Deconstructed Five-Dimensional Yang-Mills Theory*, Phys. Lett. **B532** (2002) 121, hep-ph/0201164.

[3] R.S. Chivukula, Duane A. Dicus, Hong-Jian He, *Unitarity of Compactified Five-Dimensional Yang-Mills Theory*, Phys. Lett. **B525** (2002) 175, hep-ph/0111016.

[4] C. Csaki, C. Grojean, H. Murayama, L. Pilo, J. Terning, *Gauge Theories on an Interval: Unitarity Without a Higgs*, Phys. Rev. **D69** 055006 (2004), hep-ph/0305237.

[5] C. Csaki, C. Grojean, L. Pilo, J. Terning, *Towards a Realistic Model of Higgsless Electroweak Symmetry Breaking*, Phys. Rev. Lett. **92** 101802 (2004), hep-ph/0308038.

[6] C. Csaki, C. Grojean, J. Hubisz, Y. Shirman, J. Terning, *Fermions on an Interval: Quark and Lepton Masses Without a Higgs*, hep-ph/0310355.

[7] G. Cacciapaglia, C. Csaki, C. Grojean, J. Terning, *Obliques Corrections from Higgsless Models in Warped Space*, hep-ph/0401160.

[8] Y. Nomura, *Higgsless Theory of Electroweak Symmetry Breaking from Warped Space*, JHEP **0311** (2003) 050, hep-ph/0309189.

[9] R Barbieri, A. Pomarol, R. Rattazzi, *Weakly Coupled Higgsless Theories and Precision Electroweak Tests*, Phys. Lett. **B591** (2004) 141, hep-ph/0310285.

[10] H. Davoudiasl, J.L. Hewett, B. Lillie, T.G. Rizzo, *Higgsless Electroweak Symmetry Breaking in Warped Backgrounds: Constraints and Signatures*, hep-ph/0312193

[11] G. Burdman, Y. Nomura, *Holographic Theories of Electroweak Symmetry Breaking Without a Higgs Boson*, hep-ph/0312247

[12] R. Foadi, S. Gopalakrishna, C. Schmidt, *Higgsless Electroweak Symmetry Breaking From Theory Space*, JHEP **0403** (2004) 042, hep-ph/0312324.

[13] J. Hirn, J. Stern,*The Role of Spurions in Higgsless Electroweak Effective Theories*, hep-ph/0401032.

[14] C. Schwinn, *Higgsless fermion Masses and Unitarity*, hep-ph/0402118.

[15] R.S. Chivukula, M. Kurachi, M. Tanabashi,*Generalized Weinberg Sum Rules in Deconstructed QCD* hep-ph/0403112

[16] R.S. Chivukula, H. He, J. Howard, E.H. Simmons, *The Structure of Electroweak Corrections due to Extended Gauge Symmetries*, Phys. Rev. **D69** (2004) 015009, hep-ph/0307209.

[17] R.S. Chivukula, E.H. Simmons, H. He, M. Kurachi, M. Tanabashi, *The Structure of Corrections to Electroweak Interactions in Higgsless Models*, hep-ph/0406077.

[18] R. Casalbuoni, S. De Curtis, D. Dominici, *Moose Models with vanishing S Parameter*, hep-ph/0405188.

[19] S. Gabriel, S. Nandi, G. Seidl. *6D Higgsless Standard Model*, hep-ph/0406020.

[20] T. Nagasawa, M. Sakamoto, *Higgsless Gauge Symmetry Breaking with a Large Mass Hierarchy*, hep-ph/0406024.

[21] N. Arkani-Hamed, A.G. Cohen, H. Georgi, *(De)constructing Dimesnions*, Phys. Rev. Lett. **86** (2001) 4757, hep-th/0104005

[22] C.T. Hill, S. Pokorski, J. Wang, *Gauge Invariant Effective Lagrangian For Kaluza-Klein Modes*, Phys. Rev. **D64** (2001) 105005, hep-th/0104035.

[23] RS Chivukula and H Georgi, *Composite Technicolor Standard Model*, Phys Lett **B188** (1987) 59;

[24] RS Chivukula, H Georgi and HL Randall, *A Composite Technicolor Standard Model of Quarks*, Nucl. Phys. **B292** (1987) 93.

[25] K. Sfetsos, *Dynamical Emergence of Extra Dimensions and Warped Geometries*, Nucl. Phys. **B612** (2001) 191, hep-th/0106126.